DOT/FAA/AR-99/12

Office of Aviation Research
Washington, D.C. 20591

Evaluation of the Probabilistic Design Methodology and Computer Code for Composite Structures

June 2001

Final Report

This document is available to the U.S. public through the National Technical Information Service, Springfield, Virginia 22161

U.S. Department of Transportation
Federal Aviation Administration

NOTICE

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
DOT/FAA/AR-99/12		

4. Title and Subtitle	5. Report Date
EVALUATION OF THE PROBABILISTIC DESIGN METHODOLOGY AND COMPUTER CODE FOR COMPOSITE STRUCTURES	June 2001
	6. Performing Organization Code
7. Author(s) Michael Shiao*	8. Performing Organization Report No.
9. Performing Organization Name and Address Galaxy Scientific Corporation 2500 English Creek Avenue, Building 11 Egg Harbor Township, New Jersey 08234	10. Work Unit No. (TRAIS)
	11. Contract or Grant No. DTFA03-95-D-00035
12. Sponsoring Agency Name and Address U.S. Department of Transportation Federal Aviation Administration Office of Aviation Research Washington, DC 20591	13. Type of Report and Period Covered Final
	14. Sponsoring Agency Code AIR-120

15. Supplementary Notes

*Michael Shiao is now a member of the AAR-431 section at the FAA William J. Hughes Technical Center.
The FAA William J. Hughes Technical Center Program Monitors were Donald Oplinger and Peter Shyprykevich.

16. Abstract

This report presents the results of an independent evaluation on the numerical accuracy and computational efficiency of a probabilistic design methodology for composite aircraft structures. The methodology was developed by Northrop-Grumman Commercial Aircraft Division (NGCAD) under the Federal Aviation Administration (FAA) funding through Interagency Agreement DTFA03-94-A-40021, while the associated PC-based computer code MONTE was developed through FAA Grant 96-G-0036 with the University of Texas at Arlington.

The probability calculation of NGCAD's probabilistic methodology is based on the conditional expectation method (CEM) to determine the failure probability of a specified failure event. This methodology was first verified by traditional Monte Carlo simulation (MCS) method using the computer code NESSUS developed by the National Aeronautical and Space Administration (NASA). Since Monte Carlo simulation is not efficient for small probability calculation, a mixed probabilistic method (MPM) was developed in this study to verify the results from MONTE in the 10^{-6} or less probability level. The mixed probabilistic method requires a decomposition of the overall failure function into several conditional failure functions. The probability of failure for each conditional failure function is first calculated using the CEM. The overall failure probability is then computed using the probability integration method (PIM). The mixed probabilistic method is implemented in the computer code NESSUS.

Structural reliability analyses were conducted on the wing box of a Lear Fan aircraft using the computer codes MONTE and NESSUS with MCS and MPM. The results from all the codes were compared. The comparison indicates that NGCAD probabilistic composite design methodology uses fewer random simulations than that for traditional Monte Carlo simulation method for an accurate probability prediction. The computational time was reduced by about an order of magnitude.

17. Key Words	18. Distribution Statement
Probabilistic methodology, Conditional expectation method (CEM), MONTE, NESSUS, MCS, PIM, MPM	This document is available to the public through the National Technical Information Service (NTIS), Springfield, Virginia 22161.

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	34	

Form DOT F1700.7 (8-72) Reproduction of completed page authorized

PREFACE

This report was prepared to document the results of an independent evaluation on the numerical accuracy and computational efficiency of a probabilistic design methodology for composite aircraft structures. The probabilistic design methodology and associated computer code MONTE were developed by Northrop-Grumman Commercial Aircraft Division (NGCAD) under Federal Aviation Administration (FAA) funding through Interagency Agreement DTFA03-94-A-40021.

The key FAA personnel supporting this effort were Mr. Donald Oplinger and Mr. Peter Shyprykevich. Mr. John Narciso and Mr. Mike Long of Northrop-Grumman provided valuable information in understanding and evaluating the NGCAD's probabilistic design methodology. Galaxy Scientific Corporation performed this work under Contract Number DTFA03-95-D-00035 with the FAA William J. Hughes Technical Center.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONYMS AND ABBREVIATIONS

CEM	Conditional Expectation Method
DLS	Design Limited Stress
FAA	Federal Aviation Administration
HID	Hail Induced Damage
MCS	Monte Carlo Simulation
MID	Maintenance Induced Damage
MPM	Mixed Probabilistic Method
NASA	National Aeronautics and Space Administration
NGCAD	Northrop-Grumman Commercial Aircraft Division
PIM	Probability Integration Method

EXECUTIVE SUMMARY

This report presents the results of an independent evaluation on the numerical accuracy and computational efficiency of a probabilistic design methodology for composite aircraft structures. The methodology was developed by Northrop-Grumman Commercial Aircraft Division (NGCAD) under the Federal Aviation Administration (FAA) funding through Interagency Agreement DTFA03-94-A-40021, while the associated PC-based computer code MONTE was developed through FAA Grant 96-G-0036 with the University of Texas at Arlington.

The probability calculation of NGCAD's probabilistic methodology is based on the conditional expectation method (CEM) to determine the failure probability of a specified failure event. The NGCAD methodology was first verified by comparing to the traditional Monte Carlo simulation (MCS) method using the computer code NESSUS developed by the National Aeronautics and Space Administration. Since Monte Carlo simulation is not efficient for a small probability calculation (10^{-6} or less probability level), a mixed probabilistic method (MPM) was developed in the current study to verify the results from MONTE in that range. The MPM requires a decomposition of the overall failure function into several conditional failure functions. The probability of failure for each conditional failure function is first calculated using the CEM. The overall failure probability is then computed using the probability integration method (PIM). The MPM was implemented in the computer code NESSUS to make the comparisons.

Structural reliability analyses were conducted on the wing box of a Lear Fan aircraft using the computer codes MONTE and NESSUS with MCS and MPM. The results from all the codes were compared. The comparison indicates that NGCAD probabilistic composite design methodology needs fewer random simulations than needed for the traditional Monte Carlo simulation method to obtain an accurate probability prediction. Consequently the computational time was reduced by about an order of magnitude.

1. INTRODUCTION.

Modern aircraft structures are complex assemblages of structural components that operate under load and service environments that are stochastic by nature. For safety and economic reasons, aircraft structures require durability, high reliability, light weight, high performance, and affordable cost. Composite materials are potential candidates for meeting these requirements. However, composite materials have inherently higher scatter in material properties as a result of the nature of the manufacturing process for composite materials. In order to account for various uncertainties and to satisfy design requirements, knockdown factors are used extensively to reduce allowed levels of stress in structural components. It is recognized that structures designed by deterministic methodology yield unknown risks that may pose critical problems in the future. Furthermore, the stress reductions associated with these knockdown factors and the resulting reduction in allowable design loads results in substantial weight increases without providing a quantifiable measurement of their reliability.

To properly quantify the risks associated with composite structures, a study was funded by the Federal Aviation Administration (FAA) through Interagency Agreement DTFA03-94-A-40021. Under the study, Northrop-Grumman Commercial Aircraft Division (NGCAD) developed a computational methodology for a probabilistic analysis of composite structures that may contain damage or defects [1 and 2]. The probabilistic methodology was based on the conditional expectation method (CEM) for the probability calculation of a given failure event. Under FAA Grant 95-G-0036, NGCAD developed the companion PC-based software, entitled MONTE [1].

In the discussion that follows, a major consideration is the development of an effective computational method for determining probability of failure (P_f) of a composite aircraft. It should be understood that in all subsequent discussions, P_f specifically refers to probability of failure per flight hour.

An independent evaluation of NGCAD probabilistic design methodology for composite structures was conducted by Galaxy Scientific Corporation at the FAA William J. Hughes Technical Center. The study was conducted to confirm the validity of the methodology. The NGCAD's methodology was first verified by the traditional Monte Carlo simulation (MCS) method in the computer code NESSUS developed by National Aeronautics and Space Administration [3]. Since Monte Carlo simulation is not efficient for small probability calculation, a mixed probabilistic method (MPM) was developed in the current study to verify the results from MONTE in the 10^{-6} or less probability level. The MPM requires a decomposition of the overall failure function into several conditional failure functions. After the decomposition, the probability of failure for each conditional failure function is calculated using the CEM. The overall failure probability is then computed using the probability integration method (PIM). The MPM was implemented in the computer code NESSUS to evaluate computer code MONTE.

In the evaluation, a structural reliability analysis was conducted on the wing box of a Lear Fan aircraft using the computer codes MONTE and NESSUS with MCS and MPM. Seven cases were studied to evaluate the NGCAD methodology. Comparisons of the results from all the codes were made, and the computational efficiency and accuracy that can be achieved using CEM was also investigated.

The objective and activity in each case study are briefly described here. Case 1 verified the accuracy of the Romberg numerical integration used by MONTE to determine the failure probability of each random simulation. Case 2 ensured that the gust load effect on the failure probability calculation was implemented correctly in MONTE. In Case 3, probability results were purposely set at a high failure probability level (around 10^{-4}) in order to verify MONTE prediction using the traditional Monte Carlo simulation method with single (overall) failure event.

In order to verify the failure probability prediction of MONTE in the 10^{-6} or less probability, MCS method is obvious not efficient. Therefore, more efficient probabilistic methods are needed in order to conduct the evaluation. Case 4 developed a method combining the PIM with MCS. The overall failure event was first decomposed into several conditional failure events. The probability of failure of each conditional failure event was then calculated by MCS. The overall failure probability was calculated using PIM. Case 4 confirmed that fewer random simulations are needed using PIM and MCS together than that using MCS alone.

The purpose of the Case 5 study is to verify the efficiency of CEM contained in MONTE. Therefore, Case 5 used the same conditional failure events as in Case 4. However, the probability prediction of a given conditional failure event was conducted using CEM instead of MCS used in Case 4. The method that combines PIM and CEM is referred as MPM in this report. This case study confirmed that CEM uses fewer random simulations than that for MCS.

In Case 6, the conditional failure events in Case 5 were further decomposed into subprobability events. The scatter of conditional failure probability of individual simulation in Cases 5 and 6 were compared to identify the condition under which CEM is more computationally efficient.

Studies in Cases 3 to 6 verified that MPM is a more efficient probabilistic method than MCS. Therefore MPM was used in Case 7 with an expected MONTE prediction at around the 10^{-6} probability level. Eventually, at these low probabilities of failure, the accuracy and efficiency of the computer code MONTE was verified using MPM.

2. PROBABILISTIC METHODS.

In this section, the probabilistic method CEM in NGCAD's design methodology is described in detail. Also described is the PIM. PIM is used together with CEM in order to verify the MONTE prediction in the 10^{-6} or less failure probability level. The probabilistic method that combines PIM and CEM is referred as MPM in this report.

2.1 CONDITIONAL EXPECTATION METHOD.

The NGCAD probabilistic design methodology [1 and 2] for composite structures is described in figure 1. The probabilistic algorithm of the methodology is based on CEM, which will be described in this section.

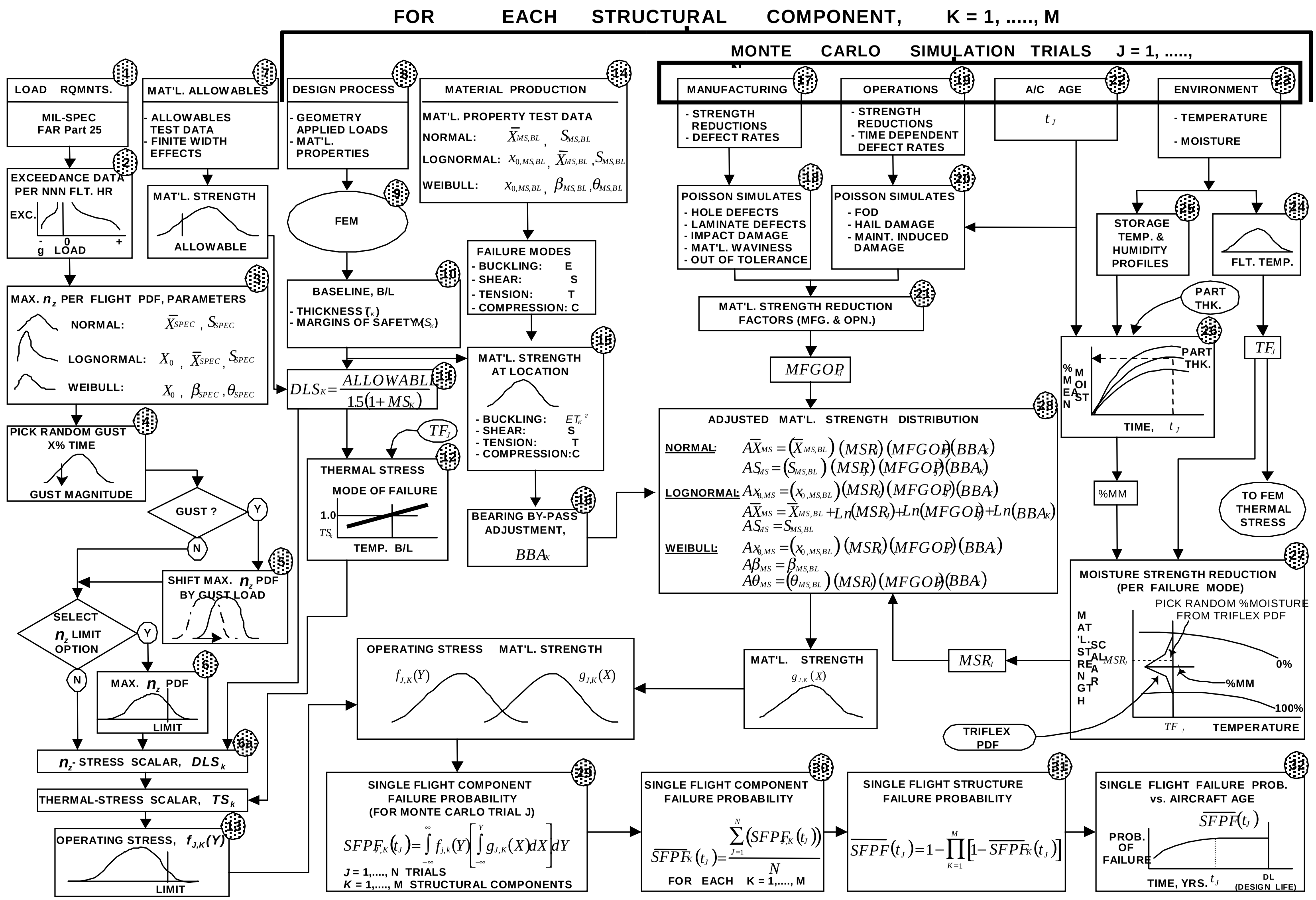

FIGURE 1. NGCAD PROBABILISTIC DESIGN MODEL

If the only random variables are strength, *S*, and load distribution, *L*, the failure probability, P_f, can be calculated from the following equations:

$$P_f = Probability(g < 0) \tag{1}$$

where

$$g = S - L$$

and

$$P_f = \int_{-\infty}^{\infty} f_L(s) F_S(s)\, ds \tag{2}$$

where F_S represents the cumulative distribution function (CDF) of strength, while f_L is the probability density function (PDF) of load. However, in real world, the random effects of gust loading, operational and manufacturing defects, temperature and moisture are superimposed on the basic load and strength distributions. In the Lear Fan case, the failure function, *g*, is defined as

$$\begin{aligned} g \;=\; & \text{strength reduction factor due to temperature and moisture} \,*\, \\ & \text{strength reduction factor due to defect} \,*\, \text{material strength} - \text{scale factor} \,*\, \\ & \text{(g load + shift factor due to gust occurrence, up gust occurrence, and} \\ & \text{up/down gust amplitude)} \end{aligned} \tag{3}$$

Failure is defined as a probability event that $g < 0$ and the probability of failure P_f is

$$P_f = Probability(g < 0) \tag{4}$$

For composite aircraft, random variables considered in the failure function *g* are temperature (*T*), moisture (*M*), gust occurrence (G_u), up gust occurrence (*U*), up gust amplitude (A_1), down gust amplitude (A_2), g load (*L*), reference material strength (*S*), five manufacturing defects and three operational defects (Q_i, i = 1, 8). Note that the probability of up gust is expressed in terms of the probability that, given the occurrence of gust, the gust is an up gust. The probability of down gust, being the complement of the probability of up gust, i.e., 1 minus its value, does not need to appear directly here. The capital letters in parentheses here represent random variables.

The probability of failure for a given probability event can always be determined using MCS by generating random samples for all random variables. However, it is well known that MCS is time consuming and often computationally impossible. Therefore, CEM approach was adopted by NGCAD to circumvent the computational demand of the MCS approach. Using CEM, the failure probability for the failure function defined in equation 3 can be computed by equation 5 below.

$$
\begin{aligned}
p_f &= \int \ldots \int P\ \ [g<0\ \mid t,m,g_u,u,a_1,a_2,q_1 \ldots q_n]\quad f_T\ f_M\ f_{G_u}\ f_U\ f_{A1}\ f_{A2}\ f_{Q_1} \ldots f_{Q_n}\ dt\ \ dm\ \ dg_u\ du\ \ da_1\, da_2\, dq_1 \ldots dq_n \\
&= \int \ldots \int P\ \ [g^c(S,L \mid t,m,g_u,u,a_1,a_2,q_1 \ldots q_n)<0]\quad f_T\ f_M\ f_{G_u}\ f_U\ f_{A1}\ f_{A2}\ f_{Q_1} \ldots f_{Q_n}\ dt\ \ dm\ \ dg_u\ du\ \ da_1\, da_2\, dq_1 \ldots dq_n \\
&= \int \ldots \int P_f^{\,c}(S,L \mid t,m,g_u,u,a_1,a_2,q_1 \ldots q_n)\quad f_T\ f_M\ f_{G_u}\ f_U\ f_{A1}\ f_{A2}\ f_{Q_1} \ldots\ f_{Q_n}\ dt\ \ dm\ \ dg_u\ du\ \ da_1\, da_2\, dq_1 \ldots dq_n \\
&= E[P_f^{\,c}]
\end{aligned}
\tag{5}
$$

In equation 5, f represents the probability density function, g represents failure function, other lowercase symbols represent realizations (randomly selected real values), P represents probability, E represents expectation, and other uppercase symbols represent random variables. Superscript c represents conditional.

The numerical procedure to calculate the probability of failure in equation 5 is described next. Monte Carlo simulation is performed repeatedly. In each simulation, a set of realizations, t, m, g_u, u, a_1, a_2, and q_1 to q_n, for random variables, T, M, G_u, U, A_1, A_2, and Q_1 to Q_n is selected. It should be pointed out that random variables of S and L are not randomly selected. Instead, a new random variable for strength, S', considering the effect of temperature, t, moisture, m, manufacturing and operational defects, q_1, to q_n, is defined [1]. Similarly, another new random variable for load, L', considering the effect of gust, g_u, u, a_1, a_2, is also defined. A conditional failure function g^c is then defined as

$$
g^c \ = \ S' - L' \tag{6}
$$

With this mathematical manipulation, the conditional failure probability P_f^c for $g^c < 0$ for each Monte Carlo simulation can be calculated by equation 2. In the NGCAD methodology, this is done through a direct numerical integration using the Romberg integration algorithm [4].

After all the desired simulations are complete, P_f is then determined by finding the average of the $P_f^{\,c}$ from all simulations. It should be noted that $P_f^{\,c}$ is a random variable itself. When $P_f^{\,c}$ is a constant value, only one MONTE simulation is needed for a converged P_f prediction. If $P_f^{\,c}$ is scattered within a narrow range, a small number of simulations will be needed. If $P_f^{\,c}$ is varied in a wider range, more simulations may be needed. In the limit, CEM becomes MCS.

2.2 PROBABILITY INTEGRATION METHOD.

As discussed in the introduction, Monte Carlo simulation method is inefficient to verify the predictions using MONTE in the 10^{-6} or less failure probability level. For example, as discussed below (section 3.3), the number of Monte Carlo simulations needed to determine a probability of failure of 10^{-6} with an error of 10% or less and a confidence of 95% is approximately $396/P_f$ or 3.96×10^8 and determination of such small probabilities of failure by standard Monte Carlo simulation becomes impractical. A more efficient probabilistic method needs to be developed. Therefore, probability integration method using conditional probabilities is used together with CEM for this purpose. The PIM approach is based on the application of the concept of conditional probability. An example is the case when the influence of gust on the load distribution is considered. In this case, the probability of failure can be computed by the following equation.

$$P_f = P_f^g * P_g + P_f^{ng} * P_{ng} \tag{7}$$

Where P_f^g is the Probability of failure with gust present and P_f^{ng} is the probability of failure without gust present; P_g is the probability of gust occurrence and P_{ng} is the probability of no gust occurrence.

Often, it is easier to determine the conditional probability of failure, given the occurrence of an event, than the direct calculation of the overall failure probability because the conditional probability of failure is always greater than the overall failure probability, so that a smaller number of simulations is needed for an accurate prediction of conditional failure probability. Based on this concept, conditional failure events are defined. The conditional failure probability for each conditional event is determined by CEM or by equivalent numerical integration algorithms. The overall failure probability is calculated by the PIM which involves the products and sums of various conditional probabilities that go to make up the overall probability of failure. This is illustrated by the examples considered in this section.

Although temperature variation on structures would normally be expected to have a continuous random distribution, the Lear Fan wing study in reference 1 treated temperature in terms of a histogram rather than a continuous distribution, so that a distribution of discrete temperatures was implied. This will affect the way temperature is treated in the following discussion.

Denote the discrete probabilities P_g, P_u, P_{qi}, and P_{tk} for various probability events as the following:

P_g = probability of gust occurrence
P_u = probability of up gust occurrence
P_{qi} = probability of occurrence of defect type i
P_{tk} = probability of occurrence of a given discrete temperature t_k

Based on the four types of events, i.e., gust occurrence, up gust occurrence, occurrence of a defect type, and occurrence of temperature, t_k, the overall failure probability can be determined by two different conditional events as follows.

2.2.1 EVENTS CONDITIONAL UPON GUST AND DEFECTS ONLY.

For the case of conditional occurrence or nonoccurrence of gust and defects, the PIM calculation of probability of failure $P(g < 0)$given in equation 4 is as follows:

$$
\begin{aligned}
P(g<0) = {} & P_g * \sum_i \left(P_{qi} * \left[(1-P_u) * P\left[g_{di}^c < 0\right] + P_u \;\; * P\left[g_{ui}^c < 0\right]\right]\right) + \\
& P_g * \left(1 - \sum_i P_{qi}\right) * \left[(1-P_u) * P\left[g_{d0}^c < 0\right] + P_u * P\left[g_{u0}^c < 0\right]\right] + \\
& (1-P_g) * \sum_i \left(P_{qi} * P\left[g_{0i}^c < 0\right]\right) + \\
& (1-P_g) * \left(1 - \sum_i P_{qi}\right) * P\left[g_{00}^c < 0\right]
\end{aligned} \tag{8}
$$

(Note the summation index i referring to the i^{th} damage type.) In equation 8, there are two subindices for each conditional event. The first is for gust occurrence, with d representing down gust occurrence, u representing up occurrence, and 0 representing no gust occurrence. The second is for defect occurrence with i representing i^{th} defect occurrence and 0 representing no defect occurrence. Letting $X(m,t)$ denote the strength reduction function and m the moisture content at temperature t, the conditional failure functions g^c are defined as follows:

g_{di}^c = $X(m,t)$* i^{th} strength reduction factor due to defects * strength - scale factor * (1+down gust amplitude) * load

g_{ui}^c = $X(m,t)$* i^{th} strength reduction factor due to defects * strength - scale factor * (1+up gust amplitude) * load

g_{d0}^c = $X(m,t)$* 1.0 * strength - scale factor *(1 + down gust amplitude) * load

g_{u0}^c = $X(m,t)$* 1.0 * strength - scale factor *(1 + up gust amplitude) * load

g_{0i}^c = $X(m,t)$* i^{th} defect strength reduction factor * strength - scale factor * load

g_{00}^c = $X(m,t)$* 1.0 * strength - scale factor * load

Random variables considered in failure functions g_{di}^c, g_{ui}^c, g_{d0}^c, and g_{u0}^c are temperature, moisture, down or up gust amplitude, g load, and material strength. Random variables considered in failure functions g_{0i}^c and g_{00}^c are temperature, moisture, g load, and material strength. Probabilities $P[g_{di}^c<0]$, $P[g_{ui}^c<0]$, $P[g_{d0}^c<0]$, $P[g_{u0}^c<0]$, $P[g_{0i}^c<0]$, and $P[g_{00}^c<0]$ are determined individually by the CEM approach, and the overall failure probability is determined by equation 8.

2.2.2 EVENTS CONDITIONAL UPON GUST, DEFECT, AND TEMPERATURE.

Since temperature is modeled by a discrete probability distribution, the failure event can be further decomposed into several more conditional failure events. The overall failure probability is then integrated by PIM as defined in equation 9.

$$P\left(g<0\right)=\sum_k P_{tk}*\left\{\begin{array}{l} P_g*\sum_i\left(P_{qi}*\left[\left(1-P_u\right)*P\left[g^c_{dik}<0\right]+P_u*P\left[g^c_{uik}<0\right]\right]\right)+ \\ P_g*\left(1-\sum_i P_{qi}\right)*\left[\left(1-P_u\right)*P\left[g^c_{d0k}<0\right]+P_u*P\left[g^c_{u0k}<0\right]\right]+ \\ \left(1-P_g\right)*\sum_i P_{qi}*P\left[g^c_{0ik}<0\right]+ \\ \left(1-P_g\right)*\left(1-\sum_i P_{qi}\right)*P\left[g^c_{00k}<0\right] \end{array}\right\} \tag{9}$$

Here a summation index k representing the k^{th} temperature is needed in addition to the summation index i representing the i^{th} damage type. Letting $X(m,t_k)$ denote the strength reduction function due to random moisture effect at temperature t_k, the conditional failure functions, g^c, are defined by the following equations. As noted, there are three subindices for each probability event, the first two representing gust and defect, respectively, as defined previously. The third index, k, represents k^{th} temperature.

$g^c{}_{dik}$ = $X(m,t_k)$* i^{th} defect reduction factor * strength - scale factor * (1 + down gust amplitude) * load

$g^c{}_{uik}$ = $X(m,t_k)$* i^{th} defect reduction factor * strength - scale factor * (1 + up gust amplitude) * load

$g^c{}_{d0k}$ = $X(m, t_k)$* 1.0 * strength - scale factor * (1 + down gust amplitude) * load

$g^c{}_{u0k}$ = $X(m, t_k)$* 1.0 * strength - scale factor * (1 + up gust amplitude) * load

$g^c{}_{0ik}$ = $X(m, t_k)$* i^{th} defect reduction factor * strength - scale factor * load

$g^c{}_{00k}$ = $X(m, t_k)$* 1.0 * strength - scale factor * load

Random variables considered in failure function $g^c{}_{dik}$, $g^c{}_{uik}$, $g^c{}_{u0k}$, and $g^c{}_{d0k}$ are moisture, up (or down) gust amplitude, g load, and material strength. Random variables considered in failure function $g^c{}_{0ik}$ and $g^c{}_{00k}$ are moisture, g load, and material strength.

It is noticed that after the decomposition of overall failure event g into subfailure events g^c, the number of variables to be randomly simulated are reduced from 12 in the event g to 1 (random moisture) in the conditional events $g^c{}_{0ik}$, $g^c{}_{00k}$, and to 2 (random moisture and random gust amplitude) in the conditional events $g^c{}_{dik}$, $g^c{}_{uik}$, $g^c{}_{u0k}$, and $g^c{}_{d0k}$. When the number of random variables in the conditional failure function is reduced, the variation of $P_f{}^c$ in CEM formulation is also reduced. Therefore, the number of simulations to obtain a converged failure probability for each decomposed failure function can be minimized. This behavior is demonstrated in Case 6.

3. CASE STUDY—VERIFICATION OF THE NGCAD ANALYSIS.

The structural reliability analysis of the Lear Fan wing box reported in reference 1 made use of the NGCAD methodology which was incorporated in the MONTE code. In this section, the results of the NGCAD analysis using MONTE are compared with those obtained using the Monte Carlo simulation module in NESSUS and the CEM module, an adaptation of CEM to the NESSUS code developed by the author. NGCAD probabilistic design methodology for composite structures was evaluated using seven case studies to be described in the following sections. The discretized temperature distribution, shown in figure 2, was applied to the Lear Fan analysis. The strength reduction factor due to temperature and moisture effects is shown in figure 3. The moisture content at a given service life and for different laminate thicknesses is shown in figure 4.

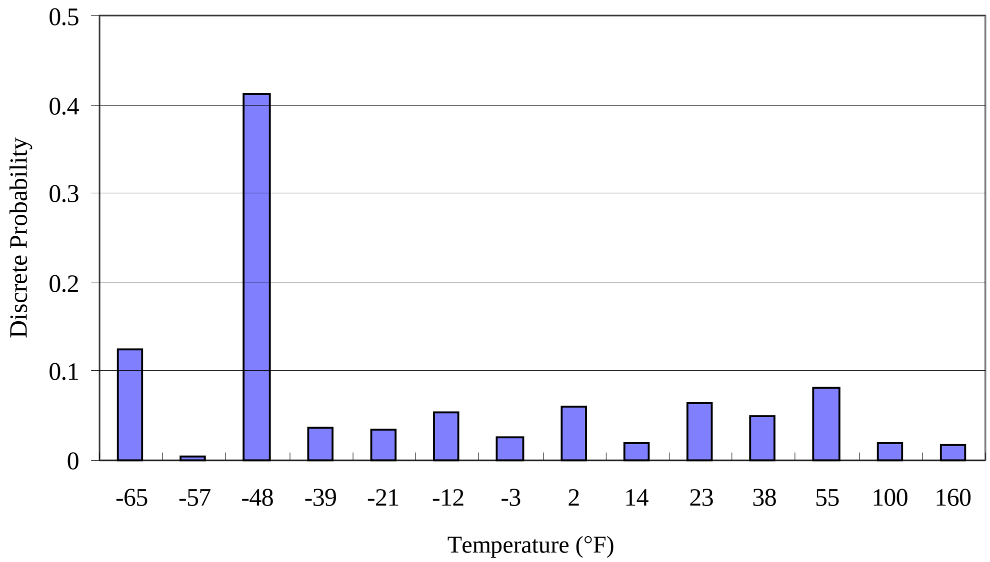

FIGURE 2. DISCRETE PROBABILITY OF RANDOM TEMPERATURE

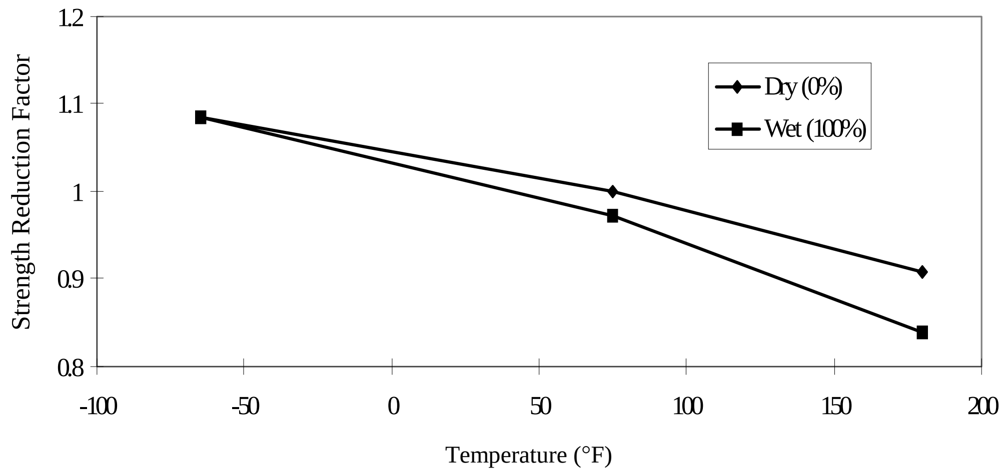

FIGURE 3. STRENGTH REDUCTION DUE TO TEMPERATURE AND MOISTURE

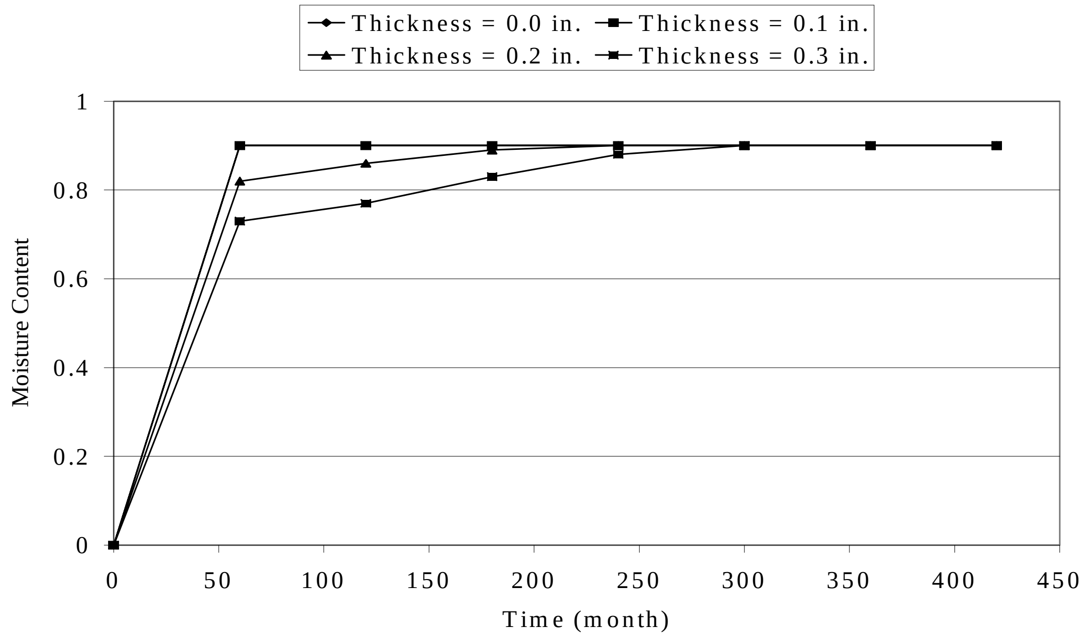

FIGURE 4. MOISTURE CONTENT CURVES

3.1 CASE 1—EVALUATION OF ROMBERG NUMERICAL INTEGRATION.

The Romberg numerical integration [4] was used in the NGCAD's MONTE code to determine the failure probability with known probability distributions for strength and stress. The Romberg numerical solution was verified using NESSUS with a simple failure function as follows.

$$\text{Failure function } g = \text{strength} - b * g \text{ load} \tag{10}$$

where

b = [scale constant]/[design factor]/[1.0 + safety margin]/ [number of g's at 100 percent design limited stress (DLS)]

The safety margin is set to be 0.68. The design factor is equal to 1.5 and the number of g's at 100 percent DLS is equal to 3.29. Manufacturing and operational defects, gust, temperature, and moisture effects were ignored in this analysis. Thus, strength and g load were the only random variables considered for this case.

Table 1 shows the probabilistic results using both NESSUS and MONTE. As shown in the table, the scale constants were varied from 10,000 to 25,000 to verify Romberg solution for failure probability ranging from 10^{-7} to 10^{-1}. It can be seen that the Romberg solution compares very well with the prediction using NESSUS at various probability levels.

TABLE 1. EVALUATION OF ROMBERG NUMERICAL INTEGRATION

Case 1	Scale Constant	P_f (NESSUS)	P_f (1 MONTE)
1-1	10,000	2.49E-07	2.56E-07
1-2	12,500	2.36E-04	2.42E-04
1-3	15,000	1.36E-03	1.39E-03
1-4	17,500	5.62E-03	5.72E-03
1-5	20,000	1.81E-02	1.83E-02
1-6	22,500	4.77E-02	4.73E-02
1-7	25,000	1.06E-01	1.06E-01

3.2 CASE 2—EVALUATION OF GUST EFFECT ON PROBABILITY PREDICTION.

Gust load effect is a complicated issue and includes random gust occurrence and random up or down gust occurrence with respective random gust amplitudes. It has a significant effect on the probabilistic prediction. The Case 2 study will verify the effect of gust on the probabilistic prediction using MONTE. In this case, the probability of defect occurrence was intentionally increased, as shown in figure 5, to obtain a relatively large failure probability. This allowed the number of random samples using both MONTE and MCS module in the NESSUS code to be reduced significantly without losing acceptable accuracy.

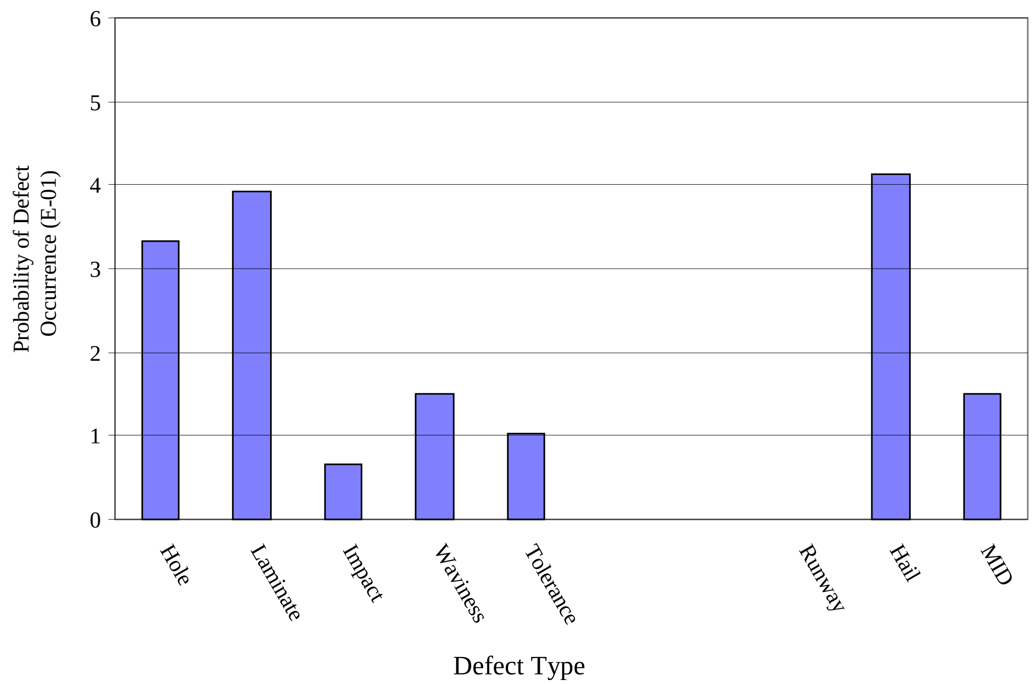

FIGURE 5. PROBABILITY OF DEFECT TYPE OCCURRENCE FOR CASE 2

Five subcases described in table 2 were studied. Cases 2-1, 2-2, and 2-3 assumed up gust, down gust, and no gust occurrence, respectively. Case 2-4 assumed gust occurrence but with different probabilities of occurrence for up and down gusts. Case 2-5 considered various combinations of gust occurrence probability and probability of up or down gust occurrence. Table 2 also shows the probabilities of failure for Cases 2-1 through 2-3 predicted by 20,000 random simulations using both MONTE and MCS in NESSUS. The results indicate good agreement. From this study, it is concluded that the method representing gust effects in MONTE is correct.

TABLE 2. EVALUATION OF GUST EFFECT ON PROBABILITY PREDICTION

Case 2	Scale Constant	Probability of Gust Occurrence	Probability of Down Gust	Probability of Up Gust	P_f (20,000 MCS)	P_f (20,000 MONTE)
2-1	10,000	1	1	0	0.0192	0.0184
2-2	10,000	1	0	1	0.0386	0.0388
2-3	10,000	0	0	0	0.0249	0.0252
2-4	10,000	1	0.2	0.8	0.0349	0.0343
2-5	10,000	0.3	0.2	0.8	0.0284	0.0279

3.3 CASE 3—VERIFICATION OF MONTE BY MCS.

In this section the predictions using MONTE for the Lear Fan wing box are compared with those using the Monte Carlo simulation method. Random variables considered in this case are *g*-load, undamaged material strength, temperature, moisture, up gust amplitude, gust occurrence, and occurrences of manufacturing and operational defects. All the random variables are generated in the Monte Carlo simulation on the basis of their respective probability distributions. All the input from the Lear Fan aircraft analysis [1], based on MONTE, was used except for the loading strain level in the composite wing. Here a strain level of 8000 μin/in is used rather than 5000, as assumed in the NGCAD Lear Fan study. The reason for this is described below.

Because the probability of failure corresponding to a loading strain level of 5000 μin/in for the Lear Fan design is less than 10^{-5}, it is difficult to verify the MONTE results using the MCS approach. This low level of failure probability can only be verified using a more efficient probabilistic approach, as will be seen in Cases 5 to 7. Thus, the strain level of 8000 μin/in was selected to increase the failure probability in order to reduce the number of MCS simulations to obtain converged results. Once a converged solution is obtained, its results can be used as a standard of comparison to judge the accuracy of other methods.

The probability of defect occurrence and corresponding strength reduction factor for each defect are plotted in figures 6 and 7. The probabilistic results predicted by both MONTE and MCS are shown in table 3. The required number of simulations for a traditional Monte Carlo method is determined based on the probability level, error bound, and confidence interval. For example, for 10% error with 95% confidence, the required number of simulations, N, is

$$N \cong 396\frac{1-P_f}{P_f} \tag{11}$$

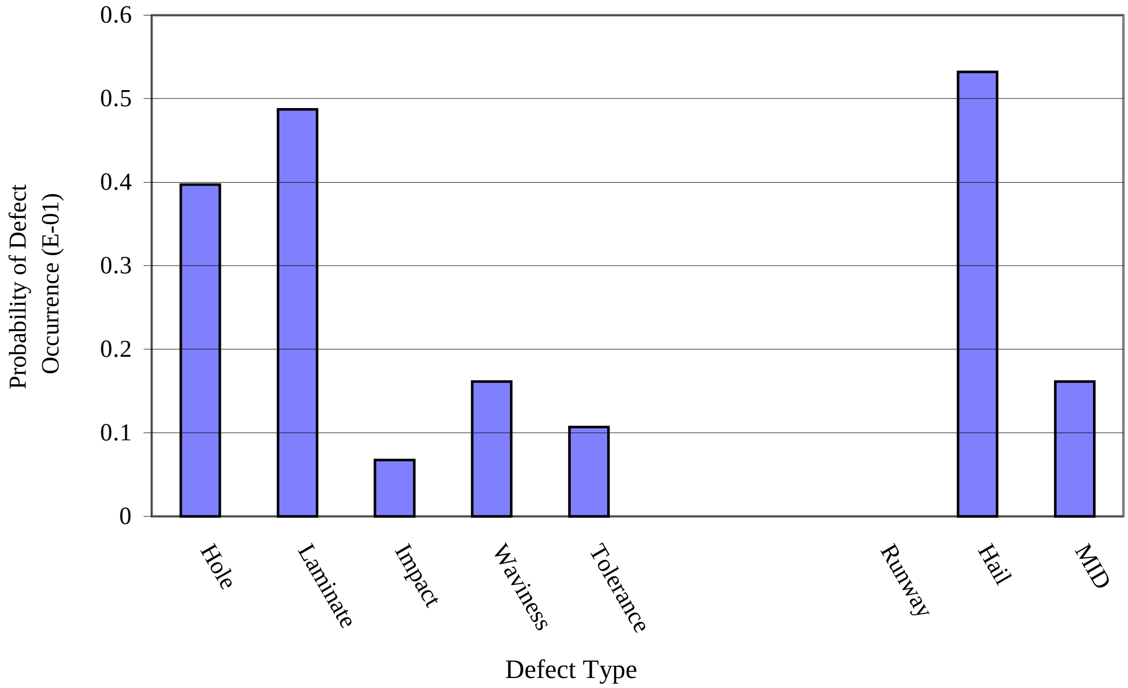

FIGURE 6. PROBABILITY OF DEFECT OCCURRENCE FOR CASE 3

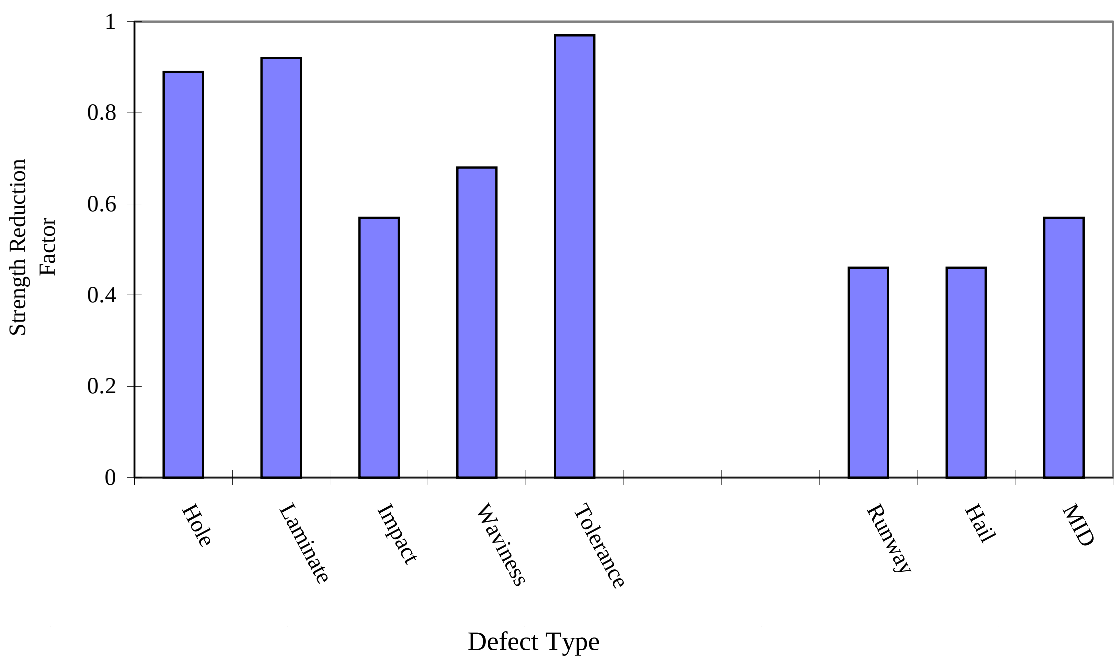

FIGURE 7. STRENGTH REDUCTION DUE TO MANUFACTURING AND OPERATIONAL DEFECTS

TABLE 3. VERIFICATION OF MONTE BY MCS

Case 3 (Thickness: 0.2 in.)	Scale Constant	P_f (2 millions MCS)	P_f (100,000 MONTE)
3-1	8,000	2.16E-04 (also see figure 8)	2.34E-04

Based on this equation, two million simulations were used for the MCS approach using NESSUS. A hundred thousand Monte Carlo simulations were used with MONTE analysis. The difference between results from both codes is within 10 percent, which indicates MONTE prediction is within desirable accuracy. The failure probability of two million simulations is also plotted in figure 8.

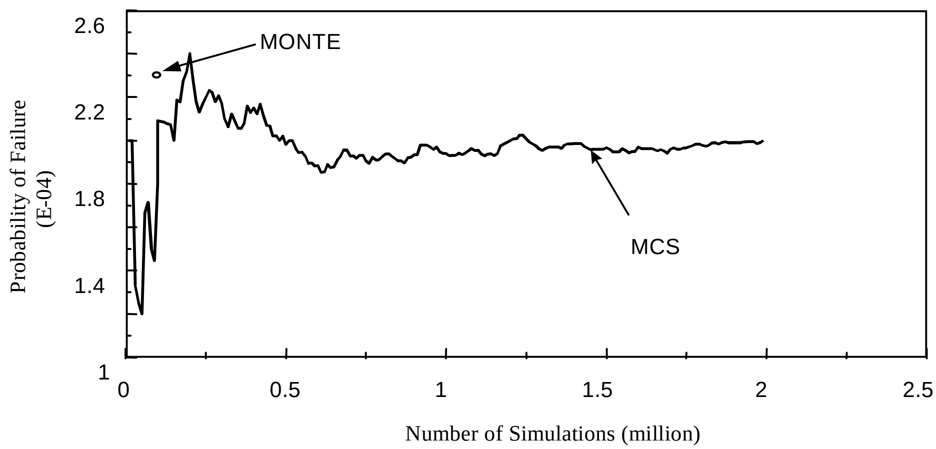

FIGURE 8. PROBABILITY OF FAILURE FOR CASE 3

Case 3 has verified the MONTE prediction at the 10^{-4} probability level using MCS alone. As discussed previously, a more efficient probabilistic method is needed for MONTE evaluation if the probability level is 10^{-6} or less. The method to be developed is a MPM using PIM and CEM together. The improvement in the computational efficiency using PIM will be studied in section 3-4. Similarly, the improvement in the computational efficiency using CEM will be studied in section 3-5.

3.4 CASE 4—EVALUATION OF THE COMPUTATIONAL EFFICIENCY OF A COMBINED PIM AND MCS PROBABILISTIC METHOD.

This study was intended to verify the improvement in computational efficiency if a combined method with PIM and MCS was used instead using MCS alone. Since a conditional probability of failure is usually larger than the overall probability of failure, fewer simulations are needed to get an accurate prediction for a conditional probability of failure. The use of this approach is described here. In this study, the overall failure event was decomposed into four major conditional probability events. In each conditional probability event, a full MCS was conducted. This means that all the random variables in each conditional probability event were randomly generated. The probability of failure of the overall failure event was determined by the probability integration shown in equation 8.

All the failure events considered are shown in table 4. Probabilistic analysis of the Lear Fan composite aircraft wing showed that failure was mainly due to hail induced defects (HID), with a small contribution (less than 5 percent) from maintenance induced defects (MID). Other defects had no impact on failure probability. Therefore, Case 4-5 considering only two types of defects, namely, HID and MID, represents approximately the same probability event as Case 3.

TABLE 4. EVALUATION OF THE COMPUTATIONAL EFFICIENCY OF A COMBINED PROBABILISTIC METHOD USING PIM AND MCS

Case 4	Scale Constant	Probability of Gust Occurrence	Probability of Up Gust	P_f (MCS)
4-1 given hail occurrence randomly generated variables: temperature and moisture	8,000	0	0	3.64E-03 (also see figure 9)
4-2 given hail occurrence randomly generated variables: temperature, moisture, and up gust amplitude	8,000	1	1	5.46E-03 (also see figure 10)
4-3 given MID occurrence randomly generated variables: temperature and moisture	8,000	0	0	4.87E-04 (also see figure 11)
4-4 given MID occurrence randomly generated variables: temperature, moisture, and up gust amplitude	8,000	1	1	6.76E-04 (also see figure 12)
4-5 randomly generated variables: gust occurrence, up gust amplitude, temperature, moisture, hail, and mid defect occurrence	8,000	0.2	1	2.16E-04 (Prediction by equation 11 using above MCS results)

For a PIM study, four conditional failure events, 4-1 to 4-4, are defined based on two types of defects, HID and MID, and two types of gust occurrence, no gust and up gust. It can be seen that the failure probabilities for Cases 4-1 and 4-2, given HID occurrence (ranging from 3.64E-03 to 5.46E-03, respectively), is more than an order of magnitude larger than the overall failure probability (2.16E-4). It means that the number of simulations is an order of magnitude fewer than that needed for the overall probability failure calculation using MCS alone. For cases 4-3 and 4-4, the associated conditional failure probability is about two to three times larger. Again, a fewer number of MCS is required.

Based on the simulation results from Cases 4-1 through 4-4, the overall failure probability for the failure event 4-5 can be predicted by the following equation:

$$\begin{aligned}
P_f \textit{ for } 4\text{-}5 = P\,(\textit{No Gust}) \quad & * \quad [P\,(\textit{HID Occurrence}) * P_f^c \textit{ for } 4\text{-}1 + \\
& \qquad P\,(\textit{MID Occurrence}) * P_f^c \textit{ for } 4\text{-}3] \; + \\
P\,(\textit{Gust Occurrence}) \; & * \quad [P\,(\textit{HID Occurrence}) * P_f^c \textit{ for } 4\text{-}2 + \\
& \qquad P\,(\textit{MID Occurrence}) * P_f^c \textit{ for } 4\text{-}4] \\
= 2.16E\text{-}04 &
\end{aligned} \tag{11}$$

where P represents probability.

The simulated results for Cases 4-1 to 4-4 are plotted in figures 9 through 12. The predicted value using equation 11 agrees well with the simulated value for Case 3-1 (2.16E-04).

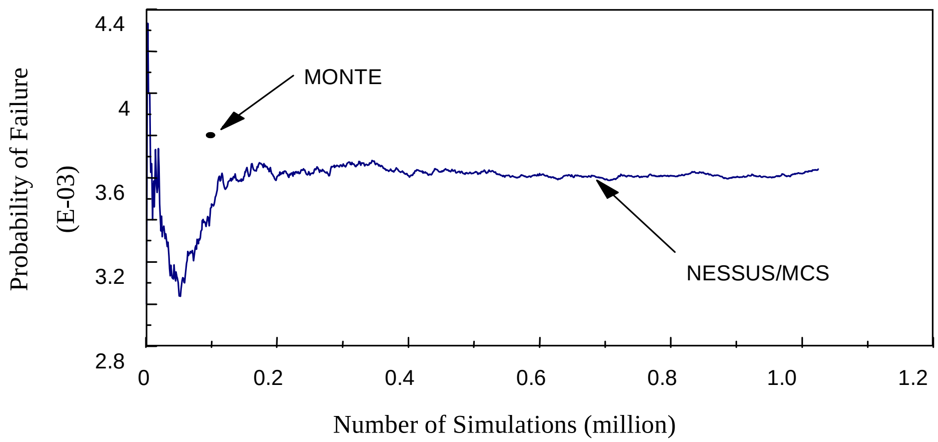

FIGURE 9. PROBABILITY OF FAILURE FOR CASE 4-1

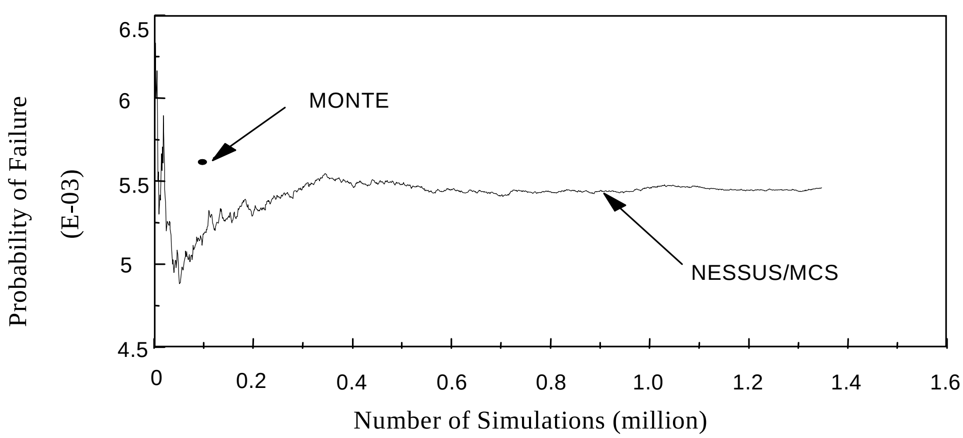

FIGURE 10. PROBABILITY OF FAILURE FOR CASE 4-2

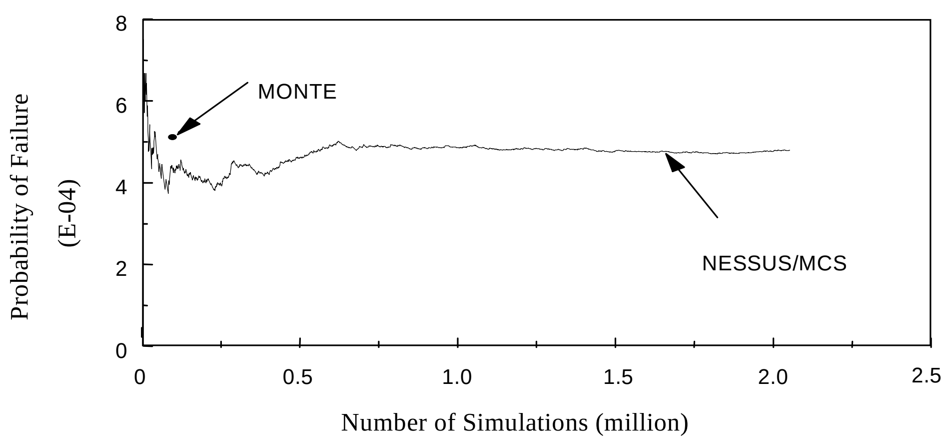

FIGURE 11. PROBABILITY OF FAILURE FOR CASE 4-3

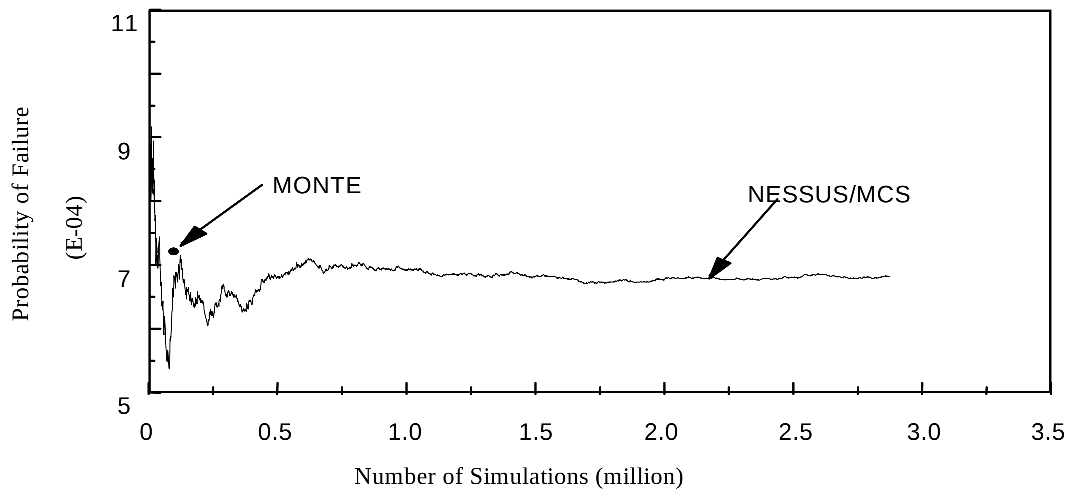

FIGURE 12. PROBABILITY OF FAILURE FOR CASE 4-4

The number of simulations for a desired accuracy in the prediction of conditional failure probability using MCS is investigated next. For Cases 4-1 and 4-2, the number of simulation is determined based on 10% error and 95% confidence. They are 0.11 and 0.07 million respectively. For Cases 4-3 and 4-4, the number of simulation is determined based on 20% error and 95% confidence. They are 0.20 and 0.14 million respectively. The reasons to use 20% error for Cases 4-3 and 4-4 are two folds: (1) probability of MID occurrence is smaller than probability of HID occurrence, and (2) $P_f^{\ c}$ for Cases 4-3 and 4-4 are an order of magnitude smaller than $P_f^{\ c}$ for Cases 4-1 and 4-2. Therefore, error bound can be increased for Cases 4-3 and 4-4 without sacrificing the accuracy of overall failure probability. From this calculation, the total number of simulations for Cases 4-1 to 4-4 is 0.52 million. As shown in table 3, the number of simulations for Case 3 using MCS approach is 2 million. As can be seen, a much smaller number of simulations are required for the combined method with PIM and MCS than that using Monte Carlo simulation alone.

3.5 CASE 5—COMPARISON OF THE COMPUTATIONAL EFFICIENCY OF CEM AND MCS.

Case 5 will investigate the improvement in computational efficiency of CEM over MCS. The conditional probability events for Case 5 are described in table 5. In this study, the failure function for Case 5-1 is the same as that in case 4-1. However, failure probability P_f for Case 5-1 is calculated using CEM, while P_f for Case 4-1 is calculated using MCS. Also in this study, the failure function for Case 5-2 is the same as that in case 4-2. However, failure probability P_f for Case 5-2 is calculated using CEM, while P_f for Case 4-2 is calculated using MCS. As discussed earlier, in order to reduce the number of simulations via CEM, certain requirements must be met. In the following, these requirements are investigated.

Case 5-1 is a conditional probability event, conditional upon the occurrence of HID, and without gust effect. Figure 13 shows the strength reduction factor for Case 5-1 due to moisture and temperature effect for each simulation in CEM. In each simulation, temperature and moisture were randomly generated based on their respective probability distribution. It can be seen that

TABLE 5. COMPARISON OF THE COMPUTATIONAL EFFICIENCY OF CEM AND MCS

Case 5	Scale Constant	Probability of Gust Occurrence	Probability of Up Gust	P_f (MPM)
5-1 given hail occurrence randomly generated variables: moisture, and temperature	8,000	0	0	3.73E-03 (also see figure 15)
5-2 given hail occurrence randomly generated variables: moisture, temperature, and up gust amplitude	8,000	1	1	5.527E-03 (also see figure 17)

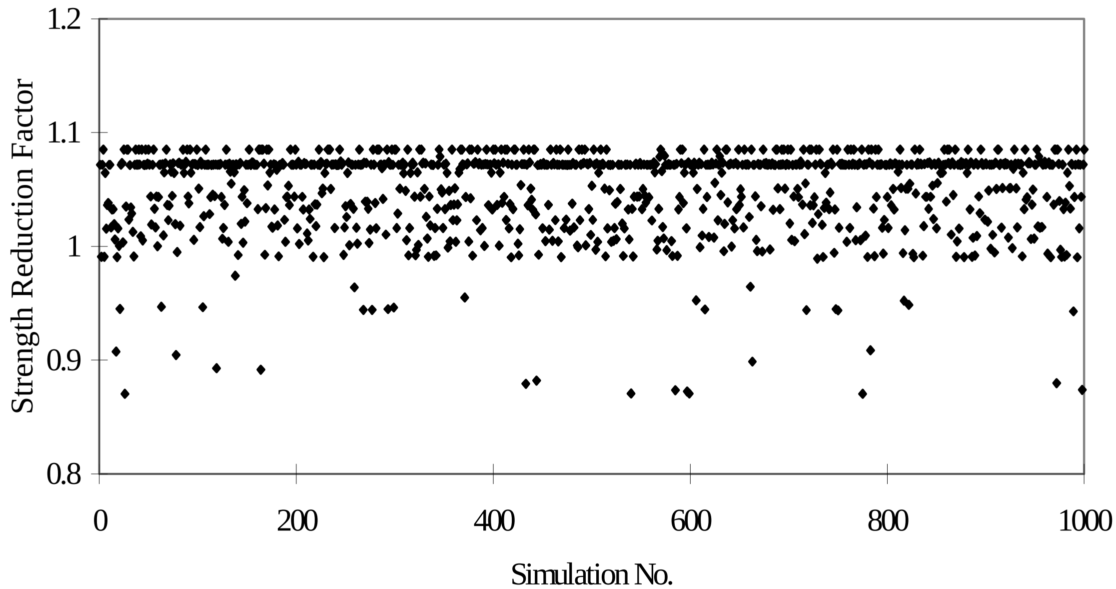

FIGURE 13. STRENGTH REDUCTION FACTOR AT EACH SIMULATION DUE TO RANDOM TEMPERATURE AND MOISTURE CONTENT

the majority of the factors are between 1.08 to 0.99. However, there are some rare occurrences falling in the range of 0.85 and 0.90. It is found that this is associated with a temperature of 160°F. Figure 14 shows the failure probability for Case 5-1 determined by equation 2 at each simulation for a set of randomly generated temperatures and moistures. Figure 14 also shows high occurrence frequency for individual failure probability in the range of 2 to 6E-03, with relatively infrequent occurrence of failure probability in the range of 0.6 to 1.8E-02.

The individual failure probability is a random variable itself. If the sample size is large enough, a probability density function can be determined. Figure 15 shows the average value of failure probability. There is not much variation after about 300 simulations.

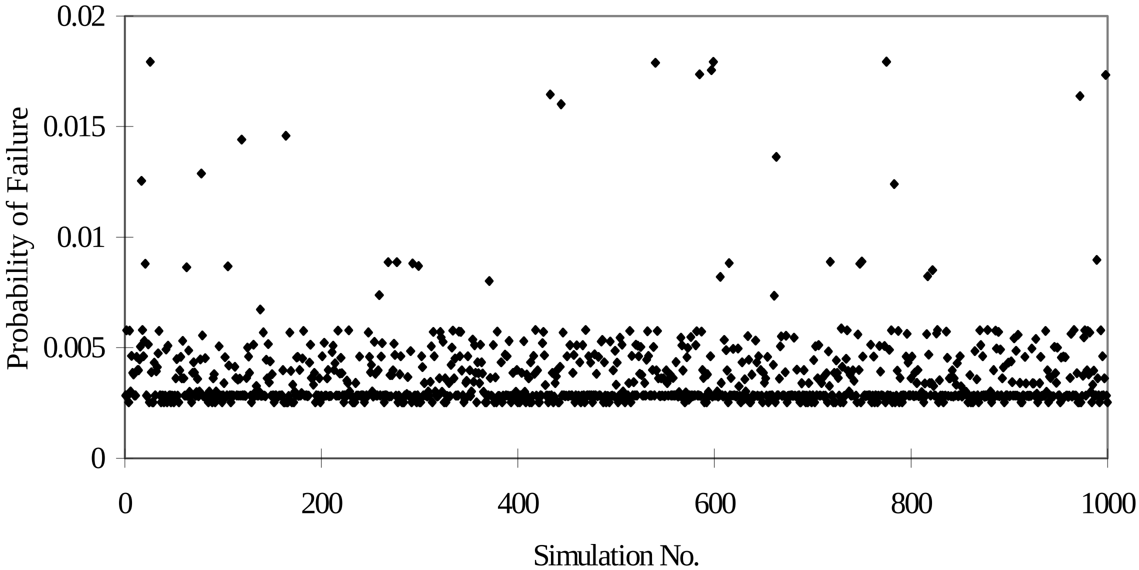

FIGURE 14. PROBABILITY OF FAILURE OF INDIVIDUAL SIMULATION FOR CASE 5-1

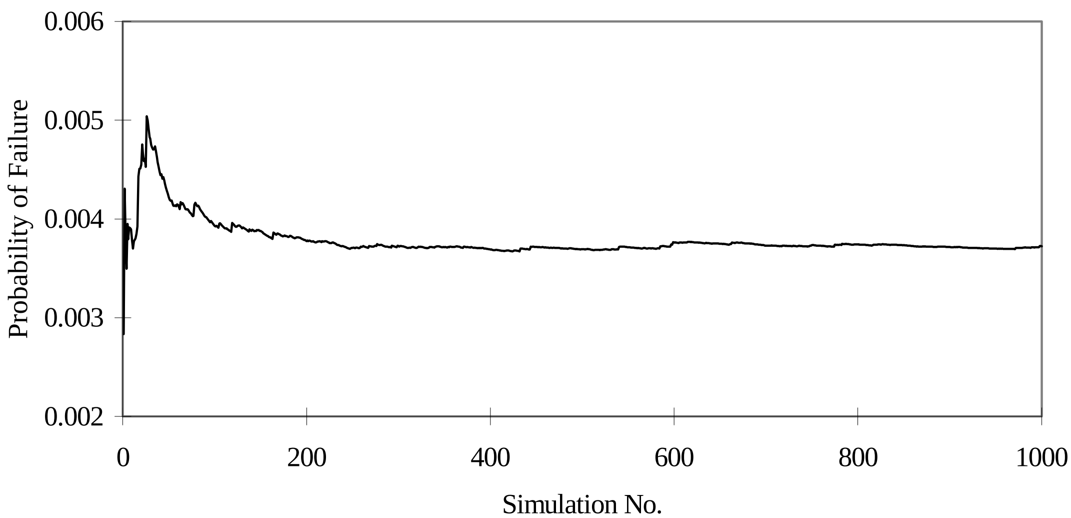

FIGURE 15. AVERAGE FAILURE PROBABILITY FOR CASE 5-1

Case 5-2 is a conditional probability event conditional upon both gust and hail defect occurrence. An additional random variable (i.e., up gust amplitude) is also randomly generated. Therefore, more simulations are expected. Figure 16 shows a high frequency for probability ranging from 3 to 10 E-03, a medium frequency for probability ranging from 1 to 1.5 E-02, and a low frequency for probability ranging from 1.5 to 3.5 E-02. As indicated in figure 17, about 500 simulations are needed for Case 5-2 versus 300 simulations needed for Case 5-1. There is only about 1 percent difference between the results from 500 simulations and the results from 1,000 simulations.

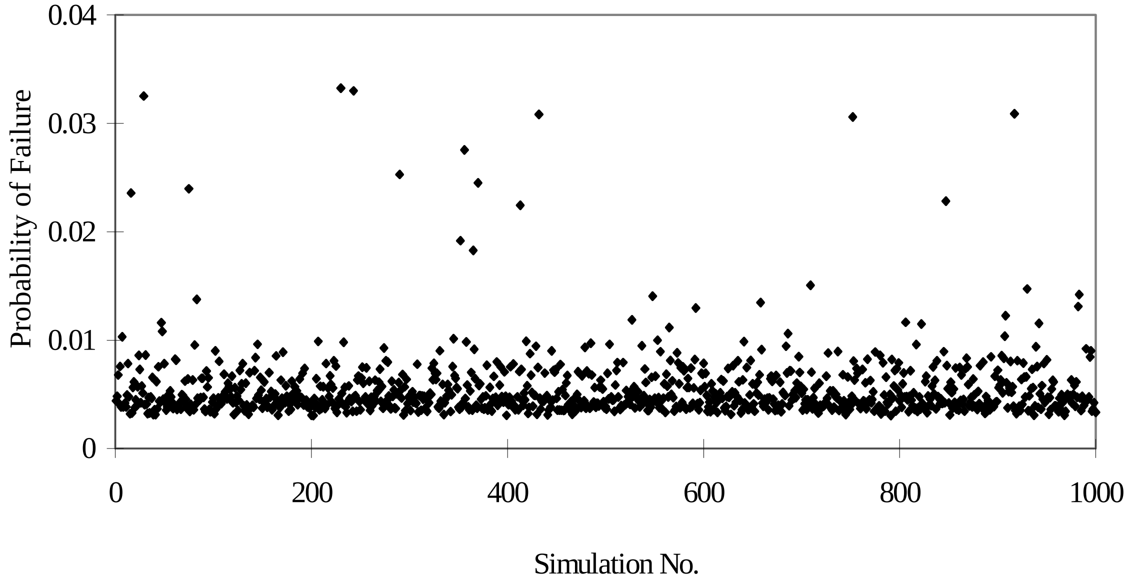

FIGURE 16. PROBABILITY OF FAILURE OF INDIVIDUAL SIMULATION FOR CASE 5-2

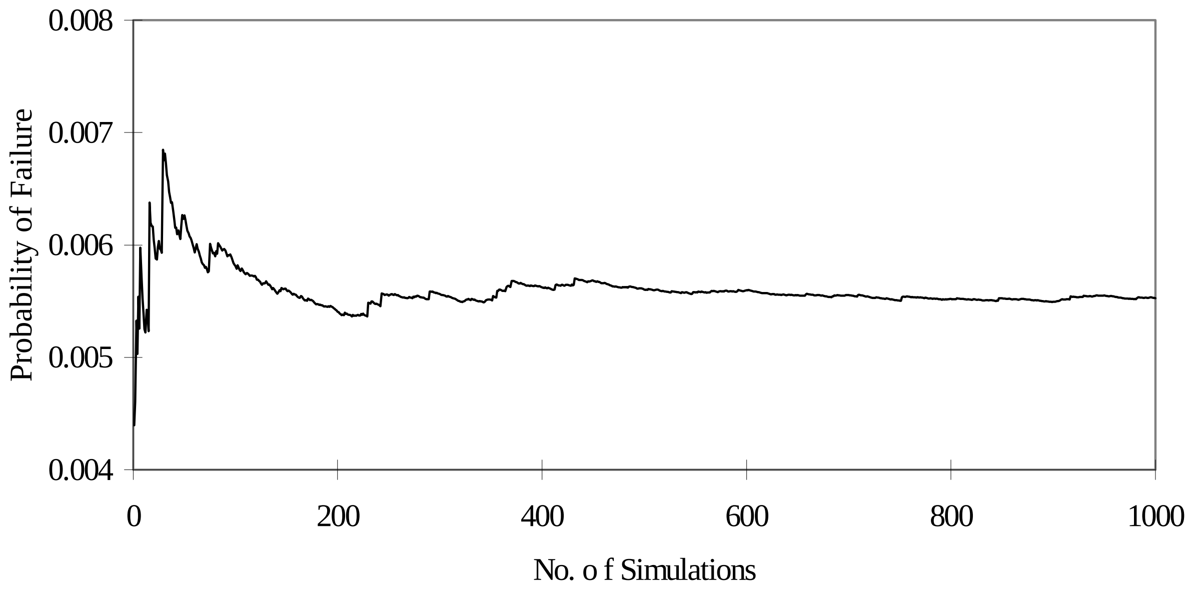

FIGURE 17. AVERAGE FAILURE PROBABILITY FOR CASE 5-2

Case 5 results, shown in table 5, compare well with results for Cases 4-1 and 4-2. As discussed in Case 4, 110 and 70 thousand simulations were required for Case 4-1 and 4-2 respectively. However, from figures 15 and 17 for Case 5-1 and 5-2 respectively, it can be seen that less than five hundred simulations are needed when CEM is used. The conclusion of this case study is that the reduction in required simulations using CEM over MCS is more than one order of magnitude.

3.6 CASE 6—DESIRABLE CONDITION FOR COMPUTATIONAL EFFICIENCY USING CEM.

In this section, the condition under which CEM is more computational efficient will be investigated. As discussed in section 2, the failure probability predicted by CEM is obtained by taking the average of individual conditional failure probability, a random variable itself. Intuitively, if the scatter of individual conditional failure probability is well behaved, fewer simulations are required to obtain a converged average value.

The probability event for Case 5-2 will be further decomposed to several subevents for the investigation. The scatter behavior of individual, conditional probability of subevent and original event will be compared. As shown in figure 2, the random temperature has a discrete probability distribution associated with 14 discrete temperatures. For each temperature, there is a corresponding probability of occurrence. Therefore, each temperature can be considered as a probability event. As a result, probability event for Case 5-2 is decomposed into 14 subprobability events conditional on HID and gust occurrence as well as a given temperature.

Since the failure probability, conditional on both defect and temperature, is one to two orders of magnitude larger than that conditional on defect only, considerably fewer samples are expected when CEM is used. As shown in figure 18, at -48°F, the probability of failure of individual simulation conditional on hail defect is uniformly scattered between 0.0033 and 0.0052. Therefore, a very small number of simulations is required to determine the expected value of the individual failure probability. As indicated in figure 19, only about 50 simulations are needed to obtain the probability of failure at -48°F, given HID occurrence.

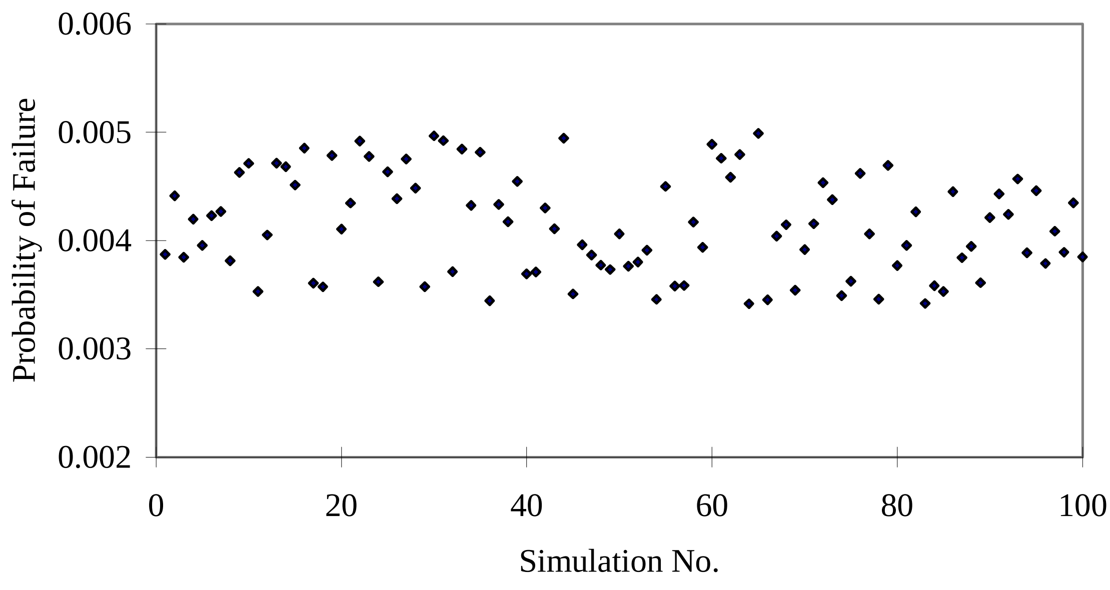

FIGURE 18. PROBABILITY OF FAILURE OF INDIVIDUAL SIMULATION AT -48°F FOR CASE 6-1

FIGURE 19. AVERAGE FAILURE PROBABILITY AT -48°F FOR CASE 6-1

A similar conclusion was also obtained for the case with 160°F, given HID occurrence, as shown in figures 20 and 21.

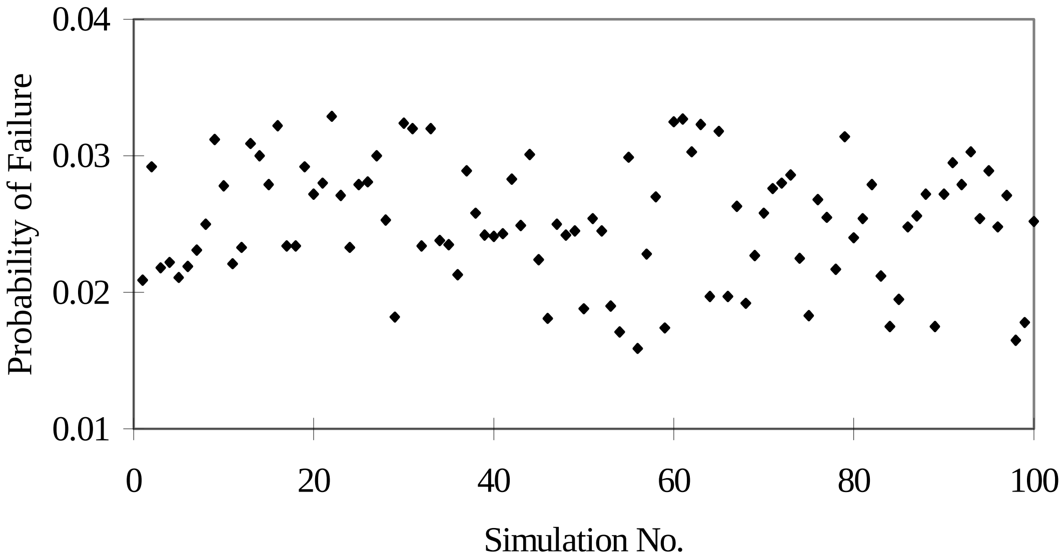

FIGURE 20. PROBABILITY OF FAILURE OF INDIVIDUAL SIMULATION AT 160°F FOR CASE 6-1

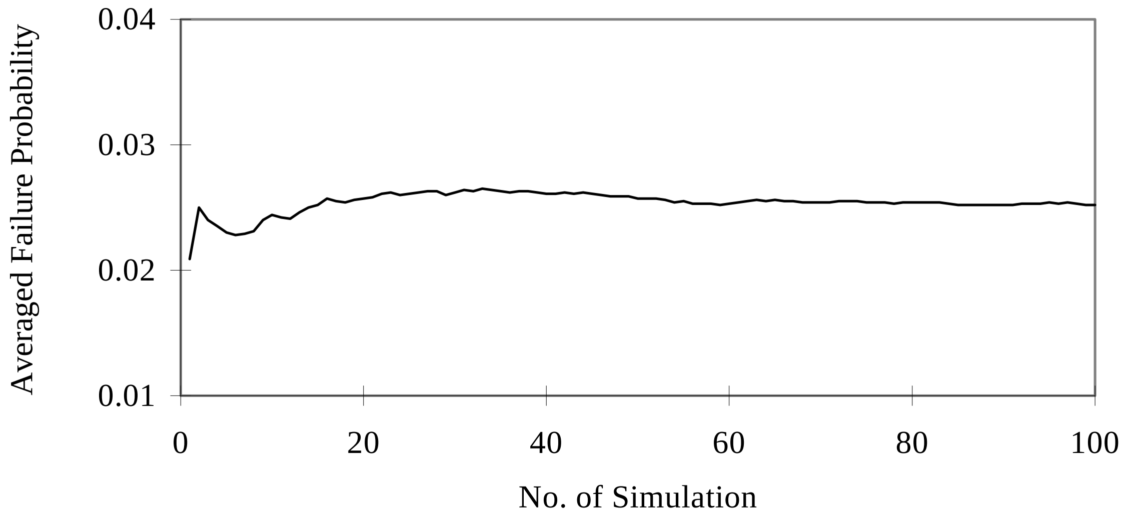

FIGURE 21. AVERAGE FAILURE PROBABILITY AT 160°F FOR CASE 6-1

In case 5-2, probability events, conditional on both gust and HID occurrence, are used for probability calculation. It can be seen from figure 16 that the scatter of the individual failure probability is wide and nonuniform. Approximately 500 simulations are needed to achieve 2% error in the conditional failure probability prediction. However, for each subprobability event in Case 6, conditional on gust and HID occurrence as well as a given temperature, individual failure probability, shown in figures 18, 20, is uniformly scattered. As a result, only 50 simulations are needed to achieve 2% error in the conditional failure probability prediction. From this observation, it is concluded that CEM will be more efficient if the individual conditional probability is uniformly scattered.

3.7 CASE 7—VERIFICATION OF MONTE BY MPM.

In Case 3, the probability of failure in the range of 10^{-4} is verified by MCS using NESSUS. However, it is extremely time consuming using traditional MCS to predict probability at a 10^{-6} probability level. In this case study, the original Lear Fan aircraft input for MONTE was used, except only HID was considered. In addition, loading strain level of 5000 μin/in was used. Subprobability events conditional upon gust, defect, and temperature, defined in section 2.2.2, are identified for a MPM analysis.

The results from MONTE were examined first for cases described in table 6. Case 7-1 uses a probability event, conditional upon hail occurrence. Noncritical random variables for random simulation include moisture content and temperature. Convergence study using MONTE was conducted using one thousand and ten thousand Monte Carlo simulations respectively. It was found the probability of failure conditional on HID occurrence can be obtained with only 1,000 MONTE simulations.

Case 7-2 has one more noncritical random variable (gust) than Case 7-1. It was found that the probability of failure, conditional on HID occurrence, can be obtained with less than 10,000 MONTE simulations. For Case 7-3, five random variables were randomly generated by

MONTE. As can be seen, converged results can be obtained with 10,000 MONTE simulations. MPM predictions are shown in table 6. Comparing the results using MONTE to that using MPM, a good agreement was found. The difference is only about 5 percent.

TABLE 6. VERIFICATION OF MONTE BY MPM

Case 7	Scale Constant	Probability of Gust Occurrence	Probability of Up Gust	P_f (MPM)	P_f (MONTE)
7-1 given hail occurrence random variables: moisture, and temperature	5,000	0	0	3.89E-05	4.10E-05(1,000 MCS) 4.10E-05(10,000 MCS)
7-2 given hail occurrence random variables: moisture, temperature, and up gust amplitude	5,000	1	1	5.14E-05	5.59E-05(1,000 MCS) 5.43E-05(10,000 MCS) 5.46E-05(100,000 MCS)
7-3 random variables: moisture, temperature, up gust amplitude, gust occurrence, and hail occurrence	5000	0.2	1	2.20E-06	2.61E-06 (1,000 MCS) 2.31E-06(10,000 MCS) 2.36E-06(100,000 MCS)

Based on this study, it is concluded that the results from MONTE are accurate for the Lear Fan aircraft analysis with 10,000 simulations. Also, the NGCAD's design methodology is efficient and accurate for the probabilistic analysis of Lear Fan aircraft composite structures.

4. CONCLUSIONS.

- NGCAD's probabilistic design methodology using the MONTE code for composite structure predicts failure probabilities that are in good agreement with Monte Carlo simulation, frequently at considerable saving in computational effort, within the range of failure probabilities ($P_f < 10^{-4}$) for which MCS can be used effectively.

- For small failure probabilities, i.e., smaller than 10^{-6}, where standard MCS cannot be used to verify MONTE because of the computational requirements, the MPM, which combines the PIM with the CEM, is an effective approach. CEM is more efficient (fewer number of simulations) when the scatter of individual, conditional failure probability from each simulation is low.

- Both the combined MCS/PIM and CEM/PIM approaches are capable of giving valid estimates of probability of failure in the low probability range corresponding to $P_f < 10^{-6}$.

- The validity of the MONTE prediction for probability of failure levels 10^{-6} or less was verified by results obtained from the mix probability method.

5. REFERENCES.

1. Gary, P.M. and Riskalla M.G., "Development of Probabilistic Design Methodology for Composite Structures," DOT/FAA/AR-95/17, August 1997.

2. Long, M. and Narcisco, J., "Probabilistic Design Methodology for Composite Aircraft Structures," DOT/FAA/AR-99/2, June 1999.

3. Probabilistic Structural Analysis Methods (PSAM) for Select Space Propulsion System Components User's Manual, Southwest Research Institute, NASA Contract NAS 3-29389, November 1989.

4. Press, F., editor, et al., Numerical Recipes FORTRAN Version, p. 114, Cambridge University Press, 1990.